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We have then the two conditions for the minimum (denoting the minimum values of p and q by p_1 and q_1),

$$2r+K=0, \qquad 2q_1+K=0; \qquad \therefore\ q_1=r. \tag{5}$$

$$\therefore\ p_1=\tfrac{1}{2}\pi r-r=0.57080r. \tag{6}$$

In the second case we have the condition

$$2rp\pi+q^2\pi=2r^2\pi \text{ or } 2rp+q^2=2r^2. \tag{7}$$

The distortion in this case is measured by the small belt left uncovered if the equatorial belt and polar caps are bent over the surface of the ball without being stretched. Call it t; then if the arc s is reckoned from the eq'r,

$$t=2r\pi\int_p^{\frac{1}{2}r\pi-q}\cos\frac{s}{r}\,ds=2r^2\pi\left(\cos\frac{q}{r}-\sin\frac{p}{r}\right), \tag{8}$$

or, introducing the condition (7),

$$\cos\frac{q}{r}-\sin\frac{p}{r}+\frac{rp+\frac{1}{2}q^2-r^2}{k^2}=\text{minimum},$$

where k is an undetermined constant.

We have then the conditions for minimum (denoting the resulting values of p and q by p_2 and q_2)

$$-\frac{1}{r}\sin\frac{q_2}{r}+\frac{q_2}{k^2}=0,\ -\frac{1}{r}\cos\frac{p_2}{r}+\frac{r}{k^2}=0;\ \therefore\ \sin\frac{q_2}{r}=\frac{q_2}{r}\cos\frac{p_2}{r}. \tag{9}$$

From (7) we have

$$\frac{p_2}{r}=1-\frac{q_2^2}{2r^2}\ldots(10,\ \therefore\ \sin\frac{q_2}{r}=\frac{q_2}{r}\cos\left(1-\frac{q_2^2}{2r^2}\right). \tag{11}$$

From these equations we find

$$q_2=0.95388r,$$
$$p_2=0.54505r.$$

The dumb-bell shaped pieces may be constructed as follows: Draw a parallelogram with the sides $r\pi$ and $2q$ the altitude of which is $2p$. On the sides $2q$ describe semicircles with q as radius; the resulting figure, together with another piece of the same shape, is the best adapted for covering a ball. In the second and all intermediate cases between the first and second case, the circular edge which is to be sewed to the straight edge is slightly shorter; the stitches have therefore to be made closer on the circular edge and in order to secure the most perfect fit it might be necessary to mark the corresponding points beforehand.

QUERY (proposed at p. 166). "How is the *Rule*, given at page 44, Gillespie's Land-surveying, (5th edition, New York, 1857,) demonstrated?"

[The rule referred to in the above query, as quoted by Dr. Oliver, is as follows:—"When the four sides and the sum of any two opposite angles are given proceed thus: Take half of the sum of the four given sides, and from it subtract each side in turn. Multiply together the four remainders, and reserve the product. Multiply together the four sides. Take half their product and multiply it by the cosine of the given sum of the angles increased by unity. Regard the sign of the cosine. Subtract this product from the reserved product and take the square root of the remainder. It will be the area of the quadrilateral."]

SOLUTION BY R. J. ADCOCK, MONMOUTH, ILL.

Let a, b, c, d be the four sides of the quadrilateral $ABCD$, $a = AB$, $b = BC$, $c = CD$, $d = AD$, and $\frac{1}{2}(a+b+c+d) = h$, then it is required to show that

$$\text{area } ABCD = \sqrt{[(h-a)(h-b)(h-c)(h-d) - \tfrac{1}{2}abcd(\cos[A+C]+1)]}.$$

Since

$$BD^2 = a^2 + d^2 + 2ad\cos A = b^2 + c^2 + 2bc\cos C, \qquad (1)$$

$$2ad\cos A - 2bc\cos C = b^2 + c^2 - a^2 - d^2,$$

and

$$a^2d^2\cos^2 A + b^2c^2\cos^2 C - 2abcd\cos A\cos C = \tfrac{1}{4}(b^2+c^2-a^2-d^2)^2. \qquad (2)$$

And since area $= \frac{1}{2}(ad\sin A + bc\sin C)$,

$$a^2d^2\sin^2 A + b^2c^2\sin^2 C + 2abcd\sin A\sin C = 4(\text{area})^2. \qquad (3)$$

Adding (2) and (3) and reducing,

$$a^2d^2 + b^2c^2 - 2abcd\cos(A+C) = 4(\text{area})^2 + \tfrac{1}{4}(b^2+c^2-a^2-d^2)^2,$$

hence

$$(\text{area})^2 = \tfrac{1}{16}[4(a^2d^2+b^2c^2)-(b^2+c^2-a^2-d^2)^2] - \tfrac{1}{2}abcd\cos(A+C)$$

$$= \tfrac{1}{16}[4a^2d^2+4b^2c^2-(b^2+c^2-a^2-d^2)^2+8abcd] - \tfrac{1}{2}abcd[1+\cos(A+C)]$$

$$= (h-a)(h-b)(h-c)(h-d) - \tfrac{1}{2}abcd[1+\cos(A+C)],$$ as required.

SOLUTIONS OF PROBLFMS IN NUMBER FOUR.

SOLUTIONS of problems in number 4 have been received as follows;

From R. J. Adcock, 173 and answer to Mr. Baker's query; Henry Gunder, 171; Henry Heaton, 170, 171 and 173; Prof E. W. Hyde, 171; G. W. Hill, 170; Chas. H. Kummell, 170, 171 and 174; Artemas Martin, 172; E. B. Seitz, 170, 171 and 173.

170. "Given the lengths of the eight edges of a quadrangular pyramid to find its altitude."